1. $\quad \begin{array}{lllllllllll}650 & 431 & 245 & 643 & 455 & 134 & 710 & 234 & 162 & 452\end{array}$
(a) The list of numbers above is to be sorted into descending order. Perform a Quick Sort to obtain the sorted list, giving the state of the list after each pass, indicating the pivot elements.

The numbers in the list represent the lengths, in mm, of some pieces of wood. The wood is sold in one metre lengths.
(b) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from the minimum number of one metre lengths. (You should ignore wastage due to cutting.)
(c) Determine whether your solution to part (b) is optimal. Give a reason for your answer.
2. A builder is asked to replace the guttering on a house. The lengths needed, in metres, are

$$
0.6,4.0,2.5,3.2,0.5,2.6,0.4,0.3,4.0 \text { and } 1.0
$$

Guttering is sold in 4 m lengths.
(a) Carry out a quick sort to produce a list of the lengths needed in descending order. You should show the result of each pass and identify your pivots clearly.
(b) Apply the first-fit decreasing bin-packing algorithm to your ordered list to determine the total number of 4 m lengths needed.
(c) Does the answer to part (b) use the minimum number of 4 m lengths? You must justify your answer.
3.
Miri
Jessie
Edward
Katie
Hegg
Beth
Louis
Philip
Natsuko
Dylan
(a) Use the quick sort algorithm to sort the above list into alphabetical order.
(b) Use the binary search algorithm to locate the name Louis.
$\begin{array}{llllllll}\text { 4. } & 52 & 48 & 50 & 45 & 64 & 47 & 53\end{array}$
The list of numbers above is to be sorted into descending order. Perform a bubble sort to obtain the sorted list, giving the state of the list after each completed pass.
(Total 4 marks)
5.

| Ali | 74 |
| :---: | :---: |
| Bobby | 28 |
| Eun-Jung | 63 |
| Katie | 54 |
| Marciana | 54 |
| Peter | 49 |
| Rory | 37 |
| Sophie | 68 |

The table shows the marks obtained by students in a test. The students are listed in alphabetical order. Carry out a quick sort to produce a list of students in descending order of marks. You should show the result of each pass and identify your pivots clearly.
6. $\quad 650431 \quad 245 \quad 643 \quad 455 \quad 134 \quad 710$
(a) The list of numbers above is to be sorted into descending order. Perform a Quick Sort to obtain the sorted list, giving the state of the list after each pass, indicating the pivot elements.

The numbers in the list represent the lengths, in mm, of some pieces of wood. The wood is sold in one metre lengths.
(b) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from the minimum number of one metre lengths. (You should ignore wastage due to cutting.)
(c) Determine whether your solution to part (b) is optimal. Give a reason for your answer.
(a) Using the quick sort algorithm, perform one complete iteration towards sorting these numbers into ascending order.
(b) Using the bubble sort algorithm, perform one complete pass towards sorting the original list into descending order.

Another list of numbers, in ascending order, is

$$
7, \quad 23, \quad 31, \quad 37, \quad 41, \quad 44, \quad 50, \quad 62, \quad 71, \quad 73, \quad 94
$$

(c) Use the binary search algorithm to locate the number 73 in this list.
8.

| 1. | Glasgow |
| :---: | :---: |
| 2. | Newcastle |
| 3. | Manchester |
| 4. | York |
| 5. | Leicester |
| 6. | Birmingham |
| 7. | Cardiff |
| 8. | Exeter |
| 9. | Southampton |
| 10. | Plymouth |

A binary search is to be performed on the names in the list above to locate the name Newcastle.
(a) Explain why a binary search cannot be performed with the list in its present form.
(b) Using an appropriate algorithm, alter the list so that a binary search can be performed. State the name of the algorithm you use.
(c) Use the binary search algorithm on your new list to locate the name Newcastle.
9. The following list gives the names of some students who have represented Britain in the International Mathematics Olympiad.

Roper ( $R$ ), Palmer ( $P$ ), Boase ( $B$ ), Young ( $Y$ ), Thomas ( $T$ ), Kenney ( $K$ ), Morris ( $M$ ), Halliwell ( $H$ ), Wicker ( $W$ ), Garesalingam ( $G$ ).
(a) Use the quick sort algorithm to sort the names above into alphabetical order.
(b) Use the binary search algorithm to locate the name Kenney.
10.
$\begin{array}{llllllll}25 & 22 & 30 & 18 & 29 & 21 & 27 & 21\end{array}$
The list of numbers above is to be sorted into descending order.
(a) (i) Perform the first pass of a bubble sort, giving the state of the list after each exchange.
(ii) Perform further passes, giving the state of the list after each pass, until the algorithm terminates.

The numbers represent the lengths, in cm, of pieces to be cut from rods of length 50 cm .
(b) (i) Show the result of applying the first fit decreasing bin packing algorithm to this situation.
(ii) Determine whether your solution to (b) (i) has used the minimum number of 50 cm rods.

1. (a) E.g:

| 650 | 431 | 245 | 643 | 455 | 710 | 234 | 162 | 452 | 134 | M1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 650 | 643 | 710 | 455 | 431 | 245 | 234 | 162 | 452 | 134 | A1 |
| 650 | 710 | 643 | 455 | 431 | 245 | 452 | 234 | 162 | 134 | A1 ft |
| 710 | 650 | 643 | 455 | 431 | 452 | 245 | 234 | 162 | 134 | A1 ft |
| 710 | 650 | 643 | 455 | 452 | 431 | 245 | 234 | 162 | 134 | A1 |

(b) $\operatorname{Bin} 1710+245$

Bin $3643+162+134$
Bin 5431 M1A1
Bin $2650+234$
Bin $4455+452$
A1A1(ft) 4
(c) $\frac{4116}{1000}=4.1165$ bins needed optimal

M1A1(ft) 2
2. (a)

| 0.6 | 4.0 | 2.5 | 3.2 | 0.5 | 2.6 | 0.4 | 0.3 | 4.0 | 1.0 | 2.6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.0 | 3.2 | 4.0 | 2.6 | 0.6 | 2.5 | 0.5 | 0.4 | 0.3 | 1.0 | 3.20 .4 | M1 |
| 4.0 | 4.0 | 3.2 | 2.6 | 0.6 | 2.5 | 0.5 | 1.0 | $\underline{0.4}$ | 0.3 | 4.00 .5 | A1 |
| 4.0 | 4.0 | 3.2 | 2.6 | 0.6 | 2.5 | 1.0 | 0.5 | 0.4 | 0.3 | 2.5 | A1ft |
| 4.0 | 4.0 | 3.2 | 2.6 | 2.5 | 0.6 | 1.0 | 0.5 | 0.4 | 0.3 | 1.0 | A1ft |
| 4.0 | 4.0 | 3.2 | 2.6 | 2.5 | 1.0 | 0.6 | 0.5 | 0.4 | 0.3 |  | A1 cso |

## Notes

1M1 Pivot, p, chosen. List sorted, >p, p. $<\mathrm{p}$ or $<\mathrm{p}, \mathrm{p},>\mathrm{p}$. If only choosing 1 pivot per iteration M1 only

1A1 $\quad 1^{\text {st }}$ pass correct and chosen next two pivots correctly for sublists $>1$

2A1ft $2^{\text {nd }}$ pass correct and chosen next two
pivots correctly for sublists $>1$
3A1ft $3^{\text {rd }}$ pass correct and next pivot for sublist $>1$ chosen correctly.

4A1 cso.

## Misread in part (a)

- If they have misread a number at the start of part (a), so genuinely miscopied and got for example 0.1 instead of 1.0 then mark the whole question as a misread - removing the last two A or B marks earned. This gives a maximum total of 9 .
- If they misread their own numbers
during the course of part (a) then count
it as an error in part (a) but mark parts
(b) and (c) as a misread. So they would lose marks in (a) for the error and then the last two A or B marks earned in (b) and (c) - giving a maximum of 8 or maybe 7 marks depending on how many marks they lose in (a).

The most popular misread is the one listed above - where 1.0 has changed to 0.1 giving
$4.04 .03 .22 .62 .50 .60 .50 .40 .3 \mathbf{0 . 1}$ at the end of (a) for this one (b) and (c) are:
(b) Length 1: 4

Length 2: 4
Length 3: $3.2 \quad 0.6$ left column \& 1.0 in place M1
$\begin{array}{llllll}\text { Length 4: } & 2.6 & 1.0 & 0.4 & 0.6 \& 0.5 & \text { A1 }\end{array}$
$\begin{array}{llllll}\text { Length 5: } & 2.5 & 0.5 & 0.3 & 0.4 & \text { A1 }\end{array}$
All correct (c.s.o) A1 4

## Note

Length 1: 4
Length 2: 4
Length 3: 3.20 .60 .1
Length 4: $2.6 \quad 0.50 .40 .3$
Length 5: 2.5
(c) $\quad 19.1 / 4=4.775$ so 5 lengths needed, accept total is 19.1 m , or refer to 0.9 'spare .

Yes, the answer to (b) does use the minimum number of bins.

DB1 2

## Note

$18.2 / 4=4.55$ so 5 bins, or total is 18.2 or 1.8 'spare'
Yes answer in (b) uses the minimum number of bins.

Alternate
Choosing middle left

| 0.6 | 4.0 | 2.5 | 3.2 | $\underline{0.5}$ | 2.6 | 0.4 | 0.3 | 4.0 | 1.0 | (pivot 0.5) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.6 | 4.0 | 2.5 | $\underline{3.2}$ | 2.6 | 4.0 | 1.0 | $\mathbf{0 . 5}$ | $\underline{0.4}$ | 0.3 | (pivots 3.2, 0.4) |
| $\underline{4.0}$ | 4.0 | $\mathbf{3 . 2}$ | 0.6 | $\underline{2.5}$ | 2.6 | 1.0 | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\underline{0.3}$ | (pivots 4.0, 2.5) |
| $\mathbf{4 . 0}$ | $\underline{4.0}$ | $\mathbf{3 . 2}$ | $\underline{2.6}$ | $\underline{\mathbf{2 . 5}}$ | $\underline{0.6}$ | 1.0 | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 3}$ | (pivot 0.6) |
| $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ | $\mathbf{3 . 2}$ | $\mathbf{2 . 6}$ | $\mathbf{2 . 5}$ | $\underline{1.0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 3}$ |  |
| $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ | $\mathbf{3 . 2}$ | $\mathbf{2 . 6}$ | $\mathbf{2 . 5}$ | $\mathbf{1 . 0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 3}$ |  |

Choosing first

| $\underline{0.6}$ | 4.0 | 2.5 | 3.2 | 0.5 | 2.6 | 0.4 | 0.3 | 4.0 | 1.0 | (pivot 0.6) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\underline{4.0}$ | 2.5 | 3.2 | 2.6 | 4.0 | 1.0 | $\mathbf{0 . 6}$ | $\underline{0.5}$ | 0.4 | 0.3 | (pivots 4.0, 0.5) |
| $\mathbf{4 . 0}$ | $\underline{2.5}$ | 3.2 | 2.6 | 4.0 | 1.0 | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\underline{0.4}$ | 0.3 | (pivots 2.5, 0.4) |
| $\mathbf{4 . 0}$ | $\underline{3.2}$ | 2.6 | 4.0 | $\mathbf{2 . 5}$ | $\underline{1.0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\underline{\mathbf{0 . 4}}$ | $\underline{0.3}$ | (pivot 3.2) |
| $\mathbf{4 . 0}$ | $\underline{4.0}$ | $\mathbf{3 . 2}$ | $\underline{2.6}$ | $\mathbf{2 . 5}$ | $\mathbf{1 . 0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 3}$ |  |
| $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ | $\mathbf{3 . 2}$ | $\mathbf{2 . 6}$ | $\mathbf{2 . 5}$ | $\mathbf{1 . 0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 3}$ |  |


| OR (alternate choosing first) |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| $\underline{0.6}$ | 4.0 | 2.5 | 3.2 | 0.5 | 2.6 | 0.4 | 0.3 | 4.0 | 1.0 | (pivot 0.6) |
| $\underline{4.0}$ | 2.5 | 3.2 | 2.6 | 4.0 | 1.0 | $\mathbf{0 . 6}$ | $\underline{0.5}$ | 0.4 | 0.3 | (pivots 4.0, 0.5) |
| $\underline{4.0}$ | $\mathbf{4 . 0}$ | $\underline{2.5}$ | 3.2 | 2.6 | 1.0 | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\underline{0.4}$ | 0.3 | (pivots 2.5, 0.4) |
| $\underline{\mathbf{4 . 0}}$ | $\mathbf{4 . 0}$ | $\underline{3.2}$ | 2.6 | $\mathbf{2 . 5}$ | $\underline{1.0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\underline{0.3}$ | (pivots 3.2) |
| $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ | $\mathbf{3 . 2}$ | $\underline{2.6}$ | $\mathbf{2 . 5}$ | $\mathbf{1 . 0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 3}$ |  |
| $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ | $\mathbf{3 . 2}$ | $\mathbf{2 . 6}$ | $\mathbf{2 . 5}$ | $\mathbf{1 . 0}$ | $\mathbf{0 . 6}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 3}$ |  |


| Sorting into ASCENDING order (full marks if then reversed, otherwise MISREAD) |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Middle left |  |  |  |  |  |  |  |  |  |  |
| 0.6 | 4.0 | 2.5 | 3.2 | $\underline{0.5}$ | 2.6 | 0.4 | 0.3 | 4.0 | 1.0 | (pivot 0.5) |
| 0.4 | 0.3 | 0.5 | 0.6 | 4.0 | 2.5 | 3.2 | 2.6 | 4.0 | 1.0 | (pivots 0.4, 3.2) |
| 0.3 | 0.4 | 0.5 | 0.6 | $\underline{2.5}$ | 2.6 | 1.0 | 3.2 | $\underline{4.0}$ | $\underline{4.0}$ | (pivots 2.5, 4.0) |
| 0.3 | 0.4 | 0.5 | 0.6 | 1.0 | 2.5 | $\underline{2.6}$ | 3.2 | 4.0 | 4.0 | (pivot 0.6) |
| 0.3 | 0.4 | 0.5 | 0.6 | 1.0 | 2.5 | 2.6 | 3.2 | 4.0 | 4.0 |  |
| Middle right |  |  |  |  |  |  |  |  |  |  |
| 0.6 | 4.0 | 2.5 | 3.2 | 0.5 | $\underline{2.6}$ | 0.4 | 0.3 | 4.0 | 1.0 | (pivot 2.6) |
| 0.6 | 2.5 | 0.5 | 0.4 | 0.3 | 1.0 | 2.6 | 4.0 | 3.2 | 4.0 | (pivots 0.4, 3.2) |
| 0.3 | 0.4 | 0.6 | 2.5 | $\underline{0.5}$ | 1.0 | 2.6 | 3.2 | 4.0 | $\underline{4.0}$ | (pivots 0.5, 4.0) |
| 0.3 | 0.4 | 0.5 | 0.6 | 2.5 | 1.0 | 2.6 | 3.2 | 4.0 | 4.0 | (pivot 2.5) |
| 0.3 | 0.4 | 0.5 | 0.6 | 1.0 | 2.5 | 2.6 | 3.2 | 4.0 | 4.0 | (pivot 1.0) |

First (1)

| $\underline{0.6}$ | 4.0 | 2.5 | 3.2 | 0.5 | 2.6 | 0.4 | 0.3 | 4.0 | 1.0 | (pivot 0.6) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\underline{0.5}$ | 0.4 | 0.3 | $\mathbf{0 . 6}$ | $\underline{4.0}$ | 2.5 | 3.2 | 2.6 | 4.0 | 1.0 | (pivots 0.5, 4.0) |
| $\underline{0.4}$ | 0.3 | $\mathbf{0 . 5}$ | $\mathbf{0 . 6}$ | $\underline{2.5}$ | 3.2 | 2.6 | 1.0 | $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ | (pivots 0.4, 2.5) |
| $\underline{0.3}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 6}$ | 1.0 | $\mathbf{2 . 5}$ | $\underline{3.2}$ | 2.6 | $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ | (pivot 3.2) |
| $\mathbf{0 . 3}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 6}$ | $\mathbf{1 . 0}$ | $\mathbf{2 . 5}$ | 2.6 | $\mathbf{3 . 2}$ | $\mathbf{4 . 0}$ | $\mathbf{4 . 0}$ |  |

First (2)

| $\underline{0.6}$ | 4.0 | 2.5 | 3.2 | 0.5 | 2.6 | 0.4 | 0.3 | 4.0 | 1.0 | (pivot 0.6) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\underline{0.5}$ | 0.4 | 0.3 | $\mathbf{0 . 6}$ | $\underline{4.0}$ | 2.5 | 3.2 | 2.6 | 4.0 | 1.0 | (pivots 0.5, 4.0) |
| $\underline{0.4}$ | 0.3 | $\mathbf{0 . 5}$ | $\mathbf{0 . 6}$ | $\underline{2.5}$ | 3.2 | 2.6 | 1.0 | 4.0 | $\mathbf{4 . 0}$ | (pivots 0.4, 2.5) |
| 0.3 | $\mathbf{0 . 4}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 6}$ | 1.0 | $\mathbf{2 . 5}$ | $\underline{3.2}$ | 2.6 | 4.0 | $\mathbf{4 . 0}$ | (pivot 3.2) |
| $\mathbf{0 . 3}$ | $\mathbf{0 . 4}$ | $\mathbf{0 . 5}$ | $\mathbf{0 . 6}$ | $\mathbf{1 . 0}$ | $\mathbf{2 . 5}$ | 2.6 | $\mathbf{3 . 2}$ | 4.0 | $\mathbf{4 . 0}$ |  |

3. (a)

| M | J | E | K | H | B | L | P | N | D | B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | M | J | E | K | H | L | P | N | D | H |
| B | E | D | H | M | J | K | L | P | N | D L |
| B | D | E | H | J | K | L | M | P | N | (E) K P |
| B | D | E | H | J | K | L | M | N | P | (J) N |
| B | D | E | H | $J$ | K | L | M | $N$ | P | (M) |

Sort completed

## Note

1M1: quick sort, pivots, p, identified, two sublists one $<$ p one $>p$.

## If choosing one pivot only per iteration,

 M1 only.1A1: first pass correct, next pivot(s) chosen consistently.

2A1ft: second pass correct, next pivot(s) chosen consistently

3A1ft: third pass correct, next pivot(s) chosen consistently

4A1: cso List re-written or end statement made or each element been chosen as a pivot.
(b) $\left[\frac{1+10}{2}\right]=6$ Katie reject left
$\left[\frac{7+10}{2}\right]=9$ Natsuko reject right
$\left[\frac{7+8}{2}\right]=8$ Miri reject right
7 = Louis name found

## Note

1M1: binary search, choosing pivot rejecting half list.

## If using unordered list then M0.

## If choosing J M1 ony

1A1: first two passes correct, condone 'sticky'pivots here, bod.

2A1ft: third pass correct, pivots rejected
3A1: cso, including success statement.

Special case
If just one letter out of order, award maximum of M1A1A0A0

| 4. e.g. | 52 | 48 | 50 | 45 | 64 | 47 | 53 | M1 |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 52 | 50 | 48 | 54 | 47 | 53 | 45 |  |  |
|  | 52 | 50 | 54 | 48 | 53 | 47 | 45 | A1 |  |
|  | 52 | 54 | 50 | 53 | 48 | 47 | 45 |  |  |
|  | 64 | 52 | 53 | 50 | 48 | 47 | 45 | A1ft |  |
|  | 64 | 53 | 52 | 50 | 48 | 47 | 45 | A1 | 4 |
|  | No further changes - list sorted |  |  |  |  |  |  |  |  |

M1 Bubble sort - $1^{\text {st }}$ pass complete - end term 64 or 45 , consistent $\mathrm{L} \rightarrow \mathrm{R}$ or $\mathrm{R} \rightarrow \mathrm{L}$ shuffle, Quick etc gets M0

A1 First 2 passes correct\} condone shrinking list
A1ft Next 2 passes correct (if $L \rightarrow R$ next pass \} condone shrinking list
A1 Final pass and final statement/rewritten list cso - must see whole list

## Notes

Bubble R $\rightarrow$ L

| 52 | 48 | 50 | 45 | 64 | 47 | 53 | M1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 64 | 52 | 48 | 50 | 45 | 53 | 47 |  |
| 64 | 53 | 52 | 48 | 50 | 45 | 47 | A1 |
| 64 | 53 | 52 | 50 | 48 | 47 | 45 | A1 |
| No further changes - list sorted |  |  |  |  |  |  |  |

## Misreads - sorting into ascending order

(note - if candidates reverse list full credit is gained.

| $\mathrm{L} \rightarrow \mathrm{R}$ (ascending - misread) MR |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 52 | 48 | 50 | 45 | 64 | 47 | 53 | M1 |
| 48 | 50 | 45 | 52 | 47 | 53 | 64 |  |
| 48 | 45 | 50 | 47 | 52 | 53 | 64 | A1 |
| 45 | 48 | 47 | 50 | 52 | 53 | 64 |  |
| 45 | 47 | 48 | 50 | 52 | 53 | 64 | A1 |
| No further changes - list sorted |  |  |  |  |  |  | A1 <br> (4-2 for misread) |

$\mathrm{R} \rightarrow \mathrm{L}$

| 52 | 48 | 50 | 45 | 64 | 47 | 53 | M1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 45 | 52 | 48 | 50 | 47 | 64 | 53 |  |
| 45 | 47 | 52 | 48 | 50 | 53 | 64 | A1 |
| 45 | 47 | 48 | 52 | 50 | 53 | 64 | A1 |
| 45 | 47 | 48 | 50 | 52 | 53 | 64 | A1 |
|  |  |  |  |  |  |  |  |
| No further changes - list sorted |  |  |  |  |  |  |  |

5. e.g

|  | 54 | M1 |
| :---: | :---: | :---: |
| $74 \quad 63$ (54) 68 54 28 (49) 37 | 5449 | A1 |
| 74 (63) 68 54 49 28 (37) | 6337 |  |
| 74 68) 63 \| 37 (28) | 68 (28) | A1ft |
| (74) 68 \| 1 | 28 |  |  |
| $\begin{array}{lllllllll}74 & 68 & 63 & 54 & 54 & 49 & 37 & 28\end{array}$ | sort complete | A1 |

$\therefore$ Ali, Sophie, Eun-Jung, \{Katie + Marciana\}, Peter, Rory, Bobby
M1 Pivot clear list > P >. Bubble sort etc. M0
A1 $1^{\text {st }}$ pass correct, next pivots correctly selected consistently

A1ft $2^{\text {nd }}+3^{\text {rd }}$ passes correct, pivots for next pan selected consistently each time. Penalise fragmented list here (or lists rewritten or all chosen as pivots)
A1 c.s.o. + stop statement (o.e.). Penalise non-sig no. errors here. Penalise "sloppyness" here
A1 c.a.o. accept c.a. even if MR

## Alternative correct answers

(i)

(ii)

| 74 | 28 | 63 | $(54)$ | 54 | 49 | 37 | 68 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 74 | 63 | 54 | 68 | 54 | 28 | $(49$ | 37 |
| 74 | 68 | 63 | 54 |  | 49 | $(28)$ | 37 |
| 74 | 68 |  | 54 |  | 1 | $(37)$ | 28 |


| 54 |  | M1 |  |
| :--- | :--- | :--- | :--- |
| 63 | 49 |  | A1 |
| 74 | 28 | (54) |  |
|  |  |  | A1ft |

(iii)


| 54 |  | M1 |  |
| ---: | ---: | ---: | ---: |
| 63 | 54 | A1 |  |
| 74 | 49 |  |  |
|  | $28(68)$ |  |  |
|  |  | $(37)$ | A1ft |

$1^{\text {st }}$ in list
(iv)


74
M1
28
63
(68) 54
(37)

Ali, Sophie, Eun-Jung, Katie + Marciana, Peter, Rory, Bobby

MISREADS ( -2 for MR)
(a)

|  |  | MR |
| :---: | :---: | :---: |
|  | 54 | M1 |
| $28 \quad 54$ (49) $37 \times 5474$ 63) 68 | 4963 | A1 |
| 28 (37) 49) (54) 63 74 (68) | 3768 (54) |  |
| (28) $37 \times 54 \mid 106$ |  | A1ft |

(b)

| $\begin{array}{llllllllll}74 & 28 & 63 & 54 & 54 & 49 & 37 & 68\end{array}$ |  |
| :---: | :---: |
| 28 (49) | 7463 (54) 68 |
| 28 (37) | (54) 74 (63) 68 |
|  | $63)$ <br> 74 <br> 68$)$ <br> 74 |

(c)

(d)


If candidates reverse list then restore full marks.
names or numbers
Bobby, Rory, Peter, Katie + Marciana, Eun-Jung, Sophie, Ali
6. (a) E.g.

M1 A1 A1ft A1ft A1 5

| 650 | 431 | 245 | 643 | (455) | 710 | 234 | 162 | 452 | 134 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 650 | (643) | 710 | 455 | 431 | 245 | (234) | 162 | 452 | 134 |
| 650 | (710) | 643 | 455 | 431 | (245) | 452 | 234 | 162 | 134 |
| 710 | 650 | 643 | 455 | 431 | (452) | 245 | 234 | 162 | 134 |
| 710 | 650 | 643 | 455 | 452 | 431 | 245 | 234 | 162 | 134 |

(b) Bin $1 \quad 710+245$

Bin $3643+162+134$
Bin 5431
M1 A1
Bin $2 \quad 650+234 \quad$ Bin $4 \quad 455+452$
A1ft A1 4
(c) eg.
$\frac{4116}{1000}=4.116 \therefore 5$ bins needed $\therefore$ optimal
M1 A1ft 2
$\begin{array}{lllllllllll}\text { 7. (a) } & \text { eg. } & 45 & 37 & 18 & 46 & 56 & 79 & 90 & 81 & 51 \\ & \text { or } & 37 & 18 & 45 & 56 & 79 & 46 & 90 & 81 & 51 \\ & \text { or } & 45 & 37 & 46 & 18 & 51 & 56 & 79 & 90 & 81\end{array}$
(b) $\quad \begin{array}{llllllllll}56 & 45 & 79 & 46 & 37 & 90 & 81 & 51 & 18\end{array}$ $\begin{array}{lllllllllllll}\text { or } & 90 & 45 & 56 & 37 & 79 & 46 & 18 & 81 & 51 & \text { M1A1 } & 2\end{array}$
(c) $\left[\frac{1+11}{2}\right]=6$ value 44 discard top
$\left[\frac{7+11}{2}\right]=9$ value 71 discard top A1
$\left[\frac{10+11}{2}\right]=11$ value 94 discard bottom
list reduces to $10^{\text {th }}$ value. This is 73 so
73 has been located as the $10^{\text {th }}$ value
A1
4
[8]
8. (a) The list is not in alphabetical order

B1 1
(b) Use of Bubble Sort or Quick Sort

M1 e.g.

## Bubble sort

| G | N | M | Y | L | B | C | E | S | P |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | G | N | M | Y | L | C | E | P | S | $1^{\text {st }}$ pass |
| B | C | G | N | M | Y | L | E | P | S | $2^{\text {nd }}$ pass |
| B | C | E | G | N | M | Y | L | P | S | $3^{\text {rd }}$ pass |
| B | C | E | G | L | N | M | Y | P | S | $4^{\text {th }}$ pass |
| B | C | E | G | L | M | N | P | Y | S | $5^{\text {th }}$ pass |
| B | C | E | G | L | M | N | P | S | Y | $6^{\text {th }}$ pass |

No more changes

## Quick sort

| G | N | M | Y |  |  | C | E | S | P |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | G | N | M | Y | (L) | C | E | S | P | $1{ }^{\text {st }}$ pass |
| B | G | C) | E | L | N | M | (Y) | S | P | $2^{\text {nd }}$ pass |
| B | C | G | (E) | L | N | M | (S) | P | Y | $3^{\text {rd }}$ pass |
| B | C | E | (G) | L | N | (M) | P | S | Y | $4^{\text {th }}$ pass |
| B | C | E | G | L | M | N | (P) | S | Y | $5^{\text {th }}$ pass |
| B | C | E | G | L | M | N | P | S | Y | $6^{\text {th }}$ pass |

No sublists > 2 and no more changes

No more changes
No sublists $>2+$ no more changes
$\begin{array}{lllllllllll}\text { (c) } & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ & \mathrm{~B} & \mathrm{C} & \mathrm{E} & \mathrm{G} & \mathrm{L} & \mathrm{M} & \mathrm{N} & \mathrm{P} & \mathrm{S} & \mathrm{Y}\end{array}$

| $\frac{[10+1]}{2}=6$ | Manchester | discard first half of list and pivot | M1 A1 |
| :--- | :--- | :--- | ---: |
| $\frac{[7+10]}{2}=9$ | Southampton | discard last half of list and pivot |  |
| $\frac{[7+8]}{2}=8$ | Plymouth | discard last half of list and pivot | A1ft |

Final term 7 Newcastle $\therefore$ word found at $7 \quad$ A1cso
4
9. (a) e.g.


5
(b) $\left[\frac{10+1}{2}\right]=6$ Palmer; reject Palmer $\rightarrow$ Young
$\left[\frac{5+1}{2}\right]=3$ Halliwell; reject Boase $\rightarrow$ Halliwell
$\left[\frac{4+5}{2}\right]=5$ Morris; reject Morris
List reduces to Kenney - name found, search complete
A1 4
[9]
10. (a)
(i)

| le ft to right |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 22 | 30 | 18 | 29 | 21 | 27 | 21 |
| 25 | 30 | 22 | 18 | 29 | 21 | 27 | 21 |
| 25 | 30 | 22 | 29 | 18 | 21 | 27 | 21 |
| 25 | 30 | 22 | 29 | 21 | 18 | 27 | 21 |
| 25 | 30 | 22 | 29 | 21 | 27 | 18 | 21 |

(ii)

| 25 | 30 | 22 | 29 | 21 | 27 | 21 | 18 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 30 | 25 | 29 | 22 | 27 | 21 | 21 | 18 |
| 30 | 29 | 25 | 27 | 22 | 21 | 21 | 18 |
| 30 | 29 | 27 | 25 | 22 | 21 | 21 | 18 |

right to le ft

| 25 | 22 | 30 | 18 | 29 | 21 | 27 | 21 | M1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 22 | 30 | 18 | 29 | 27 | 28 | 21 |  |
| 25 | 22 | 30 | 29 | 18 | 27 | 21 | 21 |  |
| 25 | 30 | 22 | 29 | 18 | 27 | 21 | 21 |  |
| 30 | 25 | 22 | 29 | 18 | 27 | 21 | 21 | A1 (pass) |
| 30 | 29 | 25 | 22 | 27 | 18 | 21 | 21 |  |
| 30 | 29 | 27 | 25 | 22 | 21 | 18 | 21 |  |
| 30 | 29 | 27 | 25 | 22 | 21 | 21 | 18 |  |
| 30 | 29 | 27 | 25 | 22 | 21 | 21 | 18 |  |


| 30 | 29 | 27 | 25 | 22 | 21 | 21 | 18 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(b) (i) $\quad$ rod $\quad 1 \quad 30 \quad 18$
$2 \quad 29 \quad 21$
32722 M1 (to the 22)
$4 \quad 25 \quad 21$
(ii) $193 \div 50=3.86, \therefore 4$ rods needed, so minimum M1 A1 2

1. No Report available for this question.
2. Many candidates scored at least 8 marks here. In part (a) a minority produced an ascending list and failed to reverse it. Some candidates did not choose their pivots consistently, swapping between middle right and middle left pivots. The decimals here caused some problems and even though the original list was printed in the answer booklet, a surprising number of candidates initially lost one item or changed one, most commonly 1.0 became 0.1 . Some candidates found only one pivot per row, with some not explicitly choosing pivots when sublists of length 2 happened to be in order - most frequently the two 4.0 s and the $1.0,0.6$ at the end. Good presentation, with a list spread evenly, in columns, across the page, helps here. (Vertical listing is rarely successful). Part (b) was generally well done, the two most popular errors being to put 0.6 in bin 5 or 0.4 in bin 5 . A significant number who had sorted the numbers into increasing order in part (a) proceeded to use a "first fit increasing" method here. In part (c) most candidates calculated the lower bound correctly. Other candidates correctly stated that since the five largest items were over half a bin in size they could not share a bin, so at least 5 bins would be needed. A few simply stated 'yes' without justification, gaining no credit.
3. This was generally well done. A disappointingly large number of candidates only chose one pivot per iteration, rather than choosing one pivot per sublist, and some candidates used lengthy methods of presentation that isolated each sublist in turn, making it difficult to see if they were choosing more than one pivot per iteration. The examiners would advise candidates to refrain from showing this unnecessary detail and simply indicate the pivots selected at each iteration. Some candidates did not select a pivot where the sublist was of order two, with the two items being in the correct order, and some did not consistently pick 'middle left' or 'middle right' when the sublist was of even order. Candidates are reminded that when the items are being transferred to the next line, the order of the items should be preserved, so if item Y is to the left of item X in the current line, neither of them being a pivot, then Y should be to the left of X in the next line. The best candidates allowed each item to become a pivot before declaring the sort complete. Some candidates did not check that their final list was in alphabetical order. In part (b) some candidates tried to apply the algorithm to the original unsorted list given at the start of (a) and others did not discard the pivot at each stage, but generally the binary search was very well done. A few candidates selected J as the first pivot, the specification makes it clear that candidates must take the 'middle right' where necessary.
4. This proved a good starter question for the candidates, with the vast majority scoring full marks. Only a few candidates sorted the list into ascending order, and very few incorrect methods were seen, but a disappointing number of candidates did not seem to be aware that a bubble sort should be performed consistently in one direction. Amongst those candidates using the correct method, more marks were lost by those misreading their own writing and changing one number into another than those lost making errors in applying the algorithm. Some candidates omitted a 'stop' statement. Candidates were asked to give the state of the list after each pass, but many showed each exchange and some each comparison, which wasted time, many of these candidates needed to use additional sheets to show all of this working and many got into time difficulties later on in the paper.
5. The vast majority of candidates showed that they understood the concept of quick sort, with very few bubble sorts seen. Most candidates chose to start with one of the 54 's as a pivot and a number of candidates were unsure what to do with the second 54 . Some chose 2 pivots initially, or created an incorrect order where the two 54s were next to each other. However, most candidates dealt well with this situation. Other common errors were: not identifying a pivot towards the end of the quick sort, where two numbers were already in the correct order, fragmenting the list rather than selecting pivots concurrently and the regularly seen re-ordering of the sub-lists. Many candidates did not produce a list of students in order.
6. Some very good answers were seen to part (a), but many candidates produced disappointing attempts. Poor presentation and lack of concentration accounted for most errors in part (a); there was inconsistent choice of pivots, numbers that disappeared from the list, numbers that mutated into other numbers and, of course, numbers being reordered in the list. A large minority sorted the list into ascending order. A number of candidates are only selecting one pivot per pass, which rather defeats the object of a quick sort. Only a very few Bubble sorts were seen. Candidate would help themselves hugely by not fixing the position of the pivots until the line after they are selected, this would avoid the need to try to cram numbers into the everdecreasing space formed by their previously chosen pivots. Candidates could then use the whole width of the line each time. Part (b) was usually well done. Some used the first fit algorithm and many put 134 into bin 5 rather than bin 3. Part (c) was often well attempted with the majority of candidates giving a clear, arithmetical argument.
7. This was generally well done. Many candidates completed the quick sort, wasting time. Some candidates did not understand the difference between an exchange and a pass in a bubble sort. Most candidates carried out the search well, but many did not give the location of the value. A large number are still assuming that the item is in the list, making statements such as 'down to one item so found'. A surprisingly large minority of candidates used the mean of the end numbers in the remaining list to create a 'pivot' which is unacceptable.
8. This question was often well answered. Most candidates correctly competed part (a), although a very few stated that the list should be in ascending rather than alphabetical order. Most could correctly name and use a suitable sorting algorithm in part (b), although some did not make their stopping statement clear and a few used a shuttle sort (not in this specification) stating that it was a bubble sort. A surprisingly large minority confused the order of the alphabet with S and P (and then M and N ) most frequently transposed. Part (c) was usually well done but candidates must make their pivots - and the order in which they select their pivots, clear. Candidates must remember to discard their pivots and note that the specification instructs them to 'round up'. Once again the stopping/found statement was sometimes missing, and some candidates assumed the presence of N , stating that once they had got down to 1 term only, that term must be N .
9. Many candidates were able to gain full marks on this question. The most common errors in part (a) were in re-ordering the letters in the sub-lists and choosing the pivots inconsistently. A surprising number of candidates seemed unsure of the alphabet. Part (b) was well done by the majority of candidates. A surprising number tried to use an unsorted list for their search, gaining no marks and others omitted to discard the pivot. The commonest error was in failing to select Morris after correctly selecting Palmer then Halliwell. A few candidates did not make the order in which they selected the pivots clear making it impossible to give credit.
10. Most candidates were able to complete the bubble sort correctly, although a number of shuttle sorts were seen from a few candidates. A number of candidates did not complete a final pass, (or stated that they had performed a final pass and found no further exchanges). The majority were able to complete the bin packing but a number were unable to show that they had used a minimum number of bins, once again the lower bound would have helped here.
